OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 4771

Decision Mathematics 1

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

Additional materials:
8 page answer booklet
Graph paper
MEI Examination Formulae and Tables (MF2)

TIME 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- There is an **insert** for use in Questions **2**, **5** and **6**.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.

This question paper consists of 7 printed pages, 1 blank page and an insert.

Section A (24 marks)

1 Table 1 shows a precedence table for a project.

Activity	Immediate predecessors	Duration (days)
A	-	5
B	-	3
C	A	3
D	A, B	4
E	A, B	5

Table 1

(i) Draw an activity-on-arc network to represent the precedences. [3]

(ii) Find the early event time and late event time for each vertex of your network, and list the critical activities. [3]

(iii) Extra resources become available which enable the durations of three activities to be reduced, each by up to two days. Which three activities should have their durations reduced so as to minimise the completion time of the project? What will be the new minimum project completion time? [2]

2 Answer this question on the insert provided.

An algorithm is specified in Fig. 2. It operates on two lists of numbers, each sorted into ascending order, to create a third list.

Step 1	Let A equal the first number in List 1. Delete the first number in List 1. Let B equal the first number in List 2. Delete the first number in List 2.
Step 2	If A $\leqslant$ B go to Step 3. Otherwise go to Step 4.
Step 3	Write A down at the end of List 3. If List 1 is not empty let A equal the first number in List 1, delete the first number in List 1 and go to Step 2. If List 1 is empty write down B at the end of List 3 and then copy the numbers in List 2 at the end of List 3. Then stop.
Step 4	Write B down at the end of List 3. If List 2 is not empty let B equal the first number in List 2, delete the first number in List 2 and go to Step 2. If List 2 is empty write down A at the end of List 3 and then copy the numbers in List 1 at the end of List 3. Then stop.

Fig. 2

(i) Complete the table in the insert showing the outcome of applying the algorithm to the following two lists:

List 1: 2, 34, 35, 56

List 2: 13, 22, 34, 81, 90, 92 [4]

(ii) What does the algorithm achieve? [1]

(iii) How many comparisons did you make in applying the algorithm? [1]

(iv) If the number of elements in List 1 is x, and the number of elements in List 2 is y, what is the maximum number of comparisons that will have to be made in applying the algorithm, and what is the minimum number? [2]

3 Fig. 3 shows a graph representing the seven bus journeys run each day between four rural towns. Each directed arc represents a single bus journey.

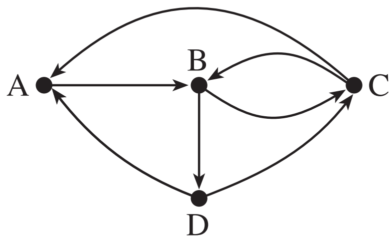

Fig. 3

(i) Show that if there is only one bus, which is in service at all times, then it must start at one town and end at a different town.

Give the start town and the end town. [3]

(ii) Show that there is only one Hamilton cycle in the graph.

Show that, if an extra journey is added from your end town to your start town, then there is still only one Hamilton cycle. [4]

(iii) A tourist is staying in town B. Give a route for her to visit every town by bus, visiting each town only once and returning to B. [1]

Section B (48 marks)

4 Table 4 shows the butter and sugar content in two recipes. The first recipe is for 1 kg of toffee and the second is for 1 kg of fudge.

	Toffee	Fudge
Butter	100 g	150 g
Sugar	800 g	700 g

Table 4

A confectioner has 1.5 kg of butter and 10 kg of sugar available. There are no constraints on the availability of other ingredients.

(i) What is the maximum amount of toffee which the confectioner could make? How much butter or sugar would be left over?

What is the maximum amount of fudge which the confectioner could make? How much butter or sugar would be left over? [4]

(ii) Formulate an LP to find the maximum total amount of toffee and fudge which the confectioner can make.

Solve your LP graphically. [8]

The confectioner charges £5.50 for 1 kg of toffee and £4.50 for 1 kg of fudge.

(iii) What quantities should he make to maximise his income? Justify your answer.

By how much would the price of toffee have to change for the maximum income solution to change? [4]

[Turn over

5 Answer this question on the insert provided.

Table 5 specifies a road network connecting 7 towns, A, B, … , G. The entries in Table 5 give the distances in miles between towns which are connected directly by roads.

	A	B	C	D	E	F	G
A	–	10	–	–	–	12	15
B	10	–	15	20	–	–	8
C	–	15	–	7	–	–	11
D	–	20	7	–	20	–	13
E	–	–	–	20	–	17	9
F	12	–	–	–	17	–	13
G	15	8	11	13	9	13	–

Table 5

(i) Using the copy of Table 5 in the insert, apply the tabular form of Prim's algorithm to the network, starting at vertex A. Show the order in which you connect the vertices.

Draw the resulting tree, give its total length and describe a practical application. [7]

(ii) The network in the insert shows the information in Table 5. Apply Dijkstra's algorithm to find the shortest route from A to E.

Give your route and its length. [6]

(iii) A tunnel is built through a hill between A and B, shortening the distance between A and B to 6 miles. How does this affect your answers to parts **(i)** and **(ii)**? [3]

6 Answer part (iv) of this question on the insert provided.

There are two types of customer who use the shop at a service station. 70% buy fuel, the other 30% do not. There is only one till in operation.

(i) Give an efficient rule for using one-digit random numbers to simulate the type of customer arriving at the service station. [1]

Table 6.1 shows the distribution of time taken at the till by customers who are buying fuel.

Time taken (mins)	1	1.5	2	2.5
Probability	$\frac{3}{10}$	$\frac{2}{5}$	$\frac{1}{5}$	$\frac{1}{10}$

Table 6.1

(ii) Specify an efficient rule for using one-digit random numbers to simulate the time taken at the till by customers purchasing fuel. [2]

Table 6.2 shows the distribution of time taken at the till by customers who are not buying fuel.

Time taken (mins)	1	1.5	2	2.5	3
Probability	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{1}{7}$

Table 6.2

(iii) Specify an efficient rule for using two-digit random numbers to simulate the time taken at the till by customers not buying fuel.

What is the advantage in using two-digit random numbers instead of one-digit random numbers in this part of the question? [4]

The table in the insert shows a partially completed simulation study of 10 customers arriving at the till.

(iv) Complete the table using the random numbers which are provided. [7]

(v) Calculate the mean total time spent queuing and paying. [2]

Candidate Name	Centre Number	Candidate Number

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Decision Mathematics 1

INSERT

Monday **23 JANUARY 2006** Afternoon 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- This **insert** should be used in Questions **2**, **5** and **6**.
- Write your name, centre number and candidate number in the spaces provided at the top of this page and attach it to your answer booklet.

This insert consists of 5 printed pages and 3 blank pages.

2 **(i)**

Step	List 1	List 2	A	B	List 3
	2, 34, 35, 56	13, 22, 34, 81, 90, 92			
1	34, 35, 56	22, 34, 81, 90, 92	2	13	
3					

(ii) ______________________________

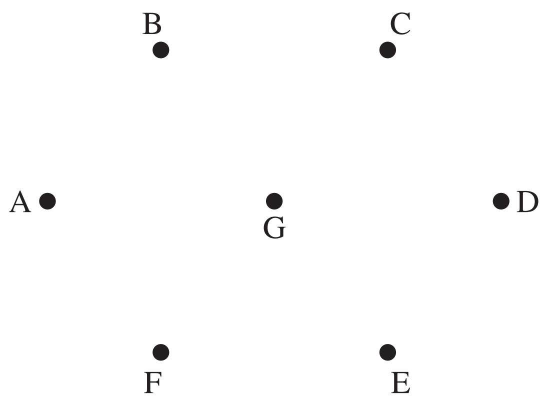
(iii) ______________________________

(iv) ______________________________

____

5 **(i)**

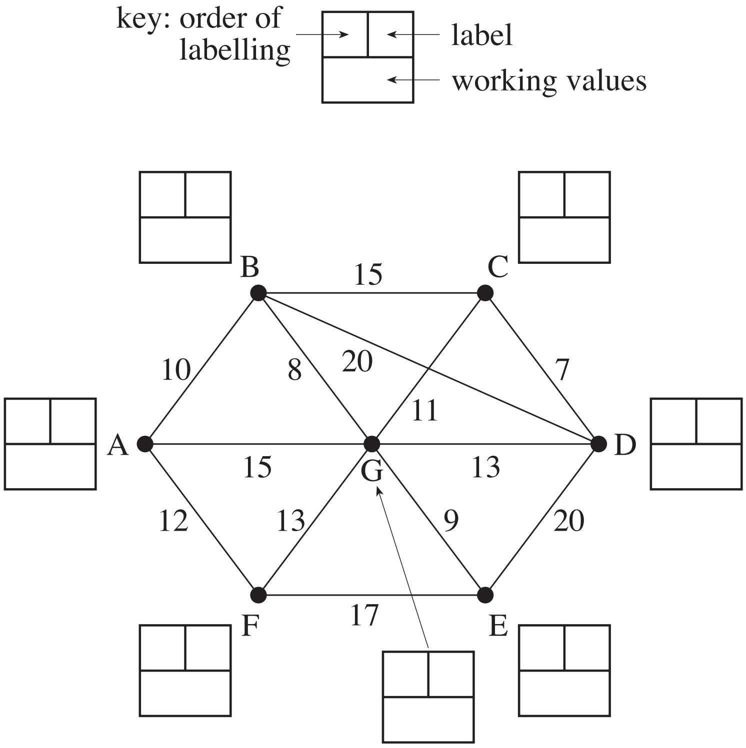
	A	B	C	D	E	F	G
A	–	10	–	–	–	12	15
B	10	–	15	20	–	–	8
C	–	15	–	7	–	–	11
D	–	20	7	–	20	–	13
E	–	–	–	20	–	17	9
F	12	–	–	–	17	–	13
G	15	8	11	13	9	13	–



Length: ________________

Application: ____

(ii)



Route: ____________________ Length: ______________________

(iii) ____

____

____

 [Turn over

6 (iv)

Random numbers for simulating type of customer:

3, 7, 8, 6, 4, 5, 1, 8, 4, 1

Random numbers for simulating time at till for customers buying fuel:

2, 4, 1, 2, 9, 8, 4, 2, 5, 4

Random numbers for simulating time at till for customers not buying fuel:

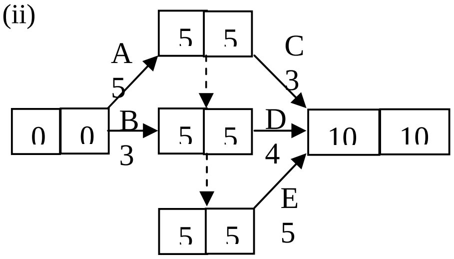
63, 32, 73, 22, 91, 45, 47, 36, 81, 52

(All times are in minutes.)

Customer number	Inter-arrival time	Arrival time in queue	Type of customer	Arrival time at till	Time taken at till	Departure time	Total time queuing and paying
1	1	1					
2	0.5						
3	3.5						
4	3						
5	1						
6	0.5						
7	1.5						
8	2						
9	2						
10	0.5						

Mark Scheme 4771
January 2006

1.

(i) & (ii) 	B1 C OK B1 D OK B1 E OK M1 early and late A1 times
Critical: A, E	B1 critical
(iii) A, E and D 6 days	B1 B1

2.

(i)

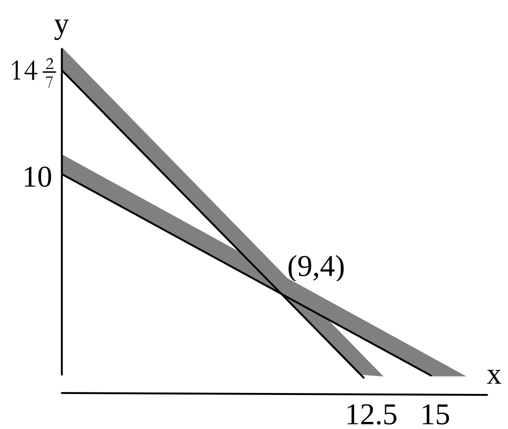
Step number	List 1	List 2	A	B	List 3
	2, 34, 35, 56	13, 22, 34, 81, 90, 92			
1	34, 35, 56	22, 34, 81, 90, 92	2	13	
3	35, 56	22, 34, 81, 90, 92	34	13	2
4	35, 56	34, 81, 90, 92	34	22	2, 13
4	35, 56	81, 90, 92	34	34	2, 13, 22
3	56	81, 90, 92	35	34	2, 13, 22, 34
4	56	90, 92	35	81	2, 13, 22, 34, 34
3		90, 92	56	81	2, 13, 22, 34, 34, 35
3		90, 92	56	81	2, 13, 22, 34, 34, 35, 56, 81, 90, 92

	M1 sca A1 to first step 3 inc. A1 to second step 3 A1 rest
(ii) Merges ordered lists to give an ordered list	B1
(iii) 7	B1
(iv) Max = x + y – 1 Min = min (x, y)	B1 B1

3.

(i)	Ins and outs One more out than in at D. Vice-versa at A. Start at D and end at A	M1 A1 B1
(ii)	Existence – A B D C A Uniqueness – Only alternative is A B C …!!! Extra arc – New possibility A D C B … !!!	B1 M1 A1 A1
(iii)	B D C A B	B1

4.

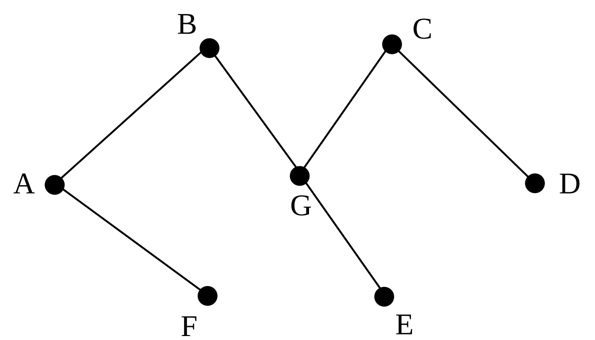
(i)	12.5 kg 250 g (of butter) 10 kg 3 kg (of sugar)	B1 B1 B1 B1
(ii)	Identification of variables e.g. Let x = kg of toffee made Let y = kg of fudge made	
	Max $x + y$ st $100x + 150y \le 1500$ $800x + 700y \le 10000$	B1 B1 B1
		B1 axes labelled and scaled B1 butter line B1 sugar line B1 shading
	Make 9 kg toffee and 4 kg fudge	B1 max x+y + solution
(iii)	12.5 kg of toffee and no fudge – either by comparing 68.75 with 67.50 with 45, or by a gradient argument	M1 A1
	Toffee price must <u>decrease</u> by £0.36, or to £5.14.	B1 B1

5.

(i)

	1 A	2 B	5 C	6 D	4 E	7 F	3 G
~~A~~	–	10	–	–	–	12	15
~~B~~	10	–	15	20	–	–	8
~~C~~	–	15	–	7	–	–	11
~~D~~	–	20	7	–	20	–	13
~~E~~	–	–		20	–	17	9
F	12	–	–	–	17	–	13
~~G~~	15	8	11	13	9	13	–

M1
A1 selections
A1 order of selecting
A1 deletions

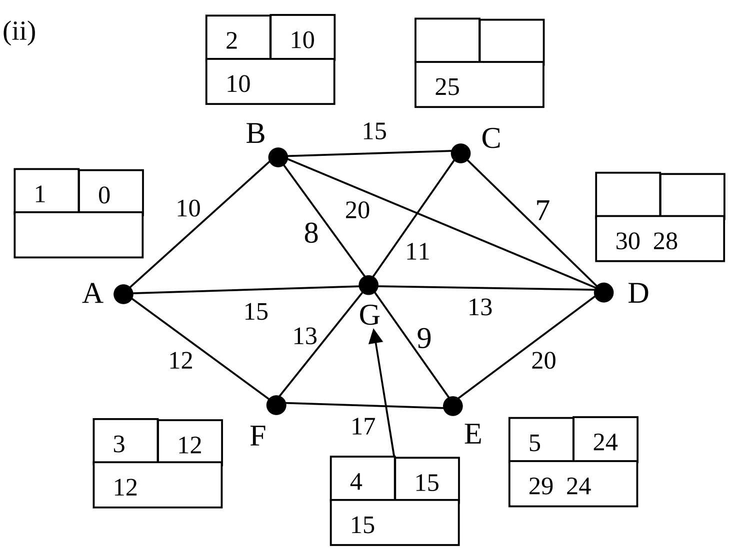


B1

Total length = 57 miles
Might be used to determine where to lay pipes or cables to connect the towns.

B1
B1

(ii)



M1 sca Dijkstra
A1 labels
A1 order of labelling
A1 working values

Shortest route: AGE
Length: 24

B1
B1

(iii) Shortens mst to 53 miles (√ by 4)
New shortest route ABGE – 23 miles (√ by 1)

B1
B1 B1

6.

(i)	e.g. 0 – 6 petrol 7 – 9 other	B1
(ii)	e.g. 0 – 2 1 min 3 – 6 1.5 mins 7 – 8 2 mins 9 2.5 mins	M1 A1
(iii)	e.g. 00 – 13 1 min 14 – 41 1.5 mins 42 – 69 2 mins 70 – 83 2.5 mins 84 – 97 3 mins 98, 99 reject	M1 some rejected A1 2 rejected A1
	Two digits – fewer rejects	B1

(iv)

Customer number	Inter-arrival time	Arrival time	Type of customer	Arrival at till	Time at till	Departure time	Queuing + paying
1	1	1	F	1	1	2	1
2	0.5	1.5	N	2	2	4	2.5
3	3.5	5	N	5	1.5	6.5	1.5
4	3	8	F	8	1.5	9.5	1.5
5	1	9	F	9.5	1	10.5	1.5
6	0.5	9.5	F	10.5	1	11.5	2
7	1.5	11	F	11.5	2.5	14	3
8	2	13	N	14	2.5	16.5	3.5
9	2	15	F	16.5	2	18.5	3.5
10	0.5	15.5	F	18.5	1.5	20	4.5

B1 arrival times
M1 types
M1 service start
M1 service duration
M1 service end
M1 time in shop
A1

(v)	24.5/10 = 2.45 mins	M1 A1

4771: Decision Mathematics 1

General Comments

Candidates at the lower end of the spectrum of ability performed broadly according to expectations. Conversely the proportion of candidates achieving marks of excellence was less than expected. This was largely due to question 3 and, to a lesser extent, question 2. In question 3 candidates often did not know about Hamilton cycles. Furthermore, very few candidates were able to mount convincing arguments where justification/proof was required. Question 2 was intricate rather than difficult, and many candidates lacked the discipline to be able to work through the algorithm. However, most candidates performed very well on the simulation question (question 6).

Comments on Individual Questions

1) **CPA**

- (i) Most candidates were able to score well here. A common error was to have activities D and E sharing the same "i" node and the same "j" node.
- (ii) Again, most candidates scored well. Some lost a mark on the backward pass by failing to take proper account of a "leaving" dummy.
- (iii) Most candidates were able to answer this correctly, even if they had made earlier errors.

2) **Algorithms**

- (i) A very large number of candidates tried to save themselves time by not writing down lists 1 to 3 at every step, but only when changes were made. This was a false economy since it resulted in confusion and error.
  It remains a mystery why a very large number of candidates miscopied their "56" from one line to "36" on the next and subsequent lines.
- (ii)(iii) Most could do part (ii). Most who completed part (i) successfully were able to do part (iii)
- (iv) By design this was, arguably, the most difficult part of the paper. Not many students were expected to get it right, and not many did.

3) **Graphs**

For the most part attempts at this question were very poor. In part (i) most candidates, but certainly not all, were able to identify the start and end nodes as D and A respectively. Very few were able to marshal an argument to prove it. In part (ii) few knew what a Hamilton cycle is, and very few who did were able to consider alternatives when it came to "showing". However, most succeeded in scoring the mark in part (iii)

4) **LP**

(i) The most common answer to the first part of this part question was to compute 1500/100=15 and 10000/800=12.5, and then to conclude that 12kg of toffee was the maximum amount which could be made.
In the second part many students correctly deduced that 10kg was the maximum amount of fudge, but then computed that 4.28kg of sugar would be left over – something to do with 10/0.7–10 ?!

(ii) The majority of candidates were able to jump through most of the hoops in part (ii). However, a substantial minority identified the variables as being the number of kg of butter and the number of kg of sugar. This led to expressions such as 100x+800y and 150x+700y – and to complete subsequent confusion.
Hardly any were able to collect the maximisation mark. They could not have this without identifying the objective and applying it. Most just found where the constraint lines intersected and quoted that point as the answer.

(iii) Many candidates came to a halt after part (ii), particularly those with butter/sugar variables. Those that did proceed mostly demonstrated that they knew what to do, even if they had not done it in part (i). The final post-optimal calculation was intended to be difficult, and few candidates scored both of the marks.

5) **Networks**

(i) (Prim – tabular form)
Most candidates were very poor at showing what they were doing. Many did not show their selections, without which it cannot be inferred that Prim is being applied. Others failed to indicate their order of node selection.
Most candidates were unable to give a practical application for a minimum connector. The majority did not seem to understand what was meant by "a practical application", and many gave as their answer the order in which they had selected arcs. Of those that did understand, few of their imagined applications could be said to be "connecting" in any way.

(ii) (Dijkstra)
This was well done, although all the usual faults were to be seen. Apart from those producing solutions without clear evidence of the use of the algorithm (especially correct working values and **only** correct working values), there were many who gave the shortest route from A to **D**.

(iii) This was very well done by most candidates.

6) **Simulation**

(i)(ii) These parts were well understood and correctly answered by most candidates. Only the
(iii) rider to part (iii) gave any difficulties, with a substantial minority of candidates apparently believing that using two-digit random numbers leads to an increase in "accuracy".

(iv)(v) Excellently done – it was a pleasure to see almost everyone doing well here.